Statistical Analysis of Pressure Relief Valve Proof Test Data: Findings and Implications

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Abstract

This paper reports on our statistical analysis of pressure relief valve (PRV) proof test data for the failure mode, fail-to-open, i.e., the PRV remains closed when actual pressure reaches or exceeds 150% of set pressure. Three data sets, from Fortune 500 operating companies, which met the intent of the quality assurance of proof test data as documented by the Center for Chemical Process Safety Process Equipment Reliability Database (CCPS PERD) initiative, were analyzed. Although the original intent of our analysis focused solely on estimation of the failure rate during the useful life of the equipment, it became apparent that the probability of failure upon initial installation or reinstallation after proof test, and the need to address what constituted end of useful life were very significant. This paper provides three important findings that are summarized as follows.

1) The statistical analysis of each data set predicted a 1% – 1.6% probability of initial failure (PIF) where initial failure is understood to be at the time of initial installation or reinstallation after a proof test. This implies that most of the failures found during the useful life via proof test are pre-existing failures from the time of installation or reinstallation rather than failures that occurred randomly after installation or reinstallation of the PRV.

2) Our calculations, based on the three independent data sets, led to consistent estimates of PRV useful-life failure rates between $10^{-8}$ and $10^{-7}$ failures/hour. Additionally, we compared our estimates from data analysis to the prediction of useful-life failure rate for a particular PRV model using the Failure Modes Effects and Diagnostics Analysis (FMEDA) method. The prediction was consistent with the data estimates.

3) The data further indicated that the low useful-life failure rate was not supported beyond a 4 to 5 year proof test interval as the threshold of wear out seemed to be approached.

The importance of these finding cannot be overestimated. When taking credit for a PRV in a risk assessment and in calculating the probability of failure on demand (PFD), both the initial probability of failing to open, as well as the probability of PRV failure due to the useful-life failure rate, must be taken into account. (The paper discusses how to do this.) Even then, the results are only defensible when it can be demonstrated that the proof test occurs before wear-out (i.e. end of useful life) begins.

Introduction

Pressure relief valves (PRV) are safety devices used extensively in the chemical process industry to reduce the risk of accidents caused by overpressure events. In order to quantify a PRV's contribution to risk reduction, it is necessary to know the device's failure modes and failure rates. This paper focuses on the PRV failure mode "fail-to-open," also known as "stuck-shut," and analyzes data to determine, for this failure mode, the PRV failure rate, useful in-service life before maintenance is required, and PIF.

The "stuck-shut" failure mode occurs when the PRV remains closed at pressures greater than or equal to 150% of set pressure. This failure mode is especially important because the PRV is normally closed. Therefore, the "stuck-shut" failure mode cannot be detected while the PRV is installed within the process being protected. Identifying this failure mode requires a proof test. In a proof test, the PRV is removed from the process and is pressurized to determine the pressure at which the PRV opens. This type of testing is costly and, while it identifies failed PRV, it does not provide an indication of when the failure occurred.
prior to the proof test – information normally needed to estimate failure rate. The nature of the proof test data requires a special form of statistical analysis known as quantal response analysis which is described in this paper.

The failure rate, $\lambda(t)$, of a PRV changes over time. Normally, risk assessment is interested in the useful-life failure rate which is approximately constant and represented by $\lambda$. In order to estimate $\lambda$ from proof test data, it is first necessary to ascertain what time interval encompasses useful life as opposed to longer time intervals when wear out is first evident and failure rates begin to rise. Thus, the first analysis described in the paper is how we obtained a useful life of 4 – 5 years. Once the useful-life interval is identified, the analysis of proof test data from this time interval is used to estimate $\lambda$. Both the full data analysis and the useful-life data analysis give estimates for the PIF. Finally, we address how PIF is incorporated into the calculation of PFD and PFDavg. We close with a discussion of the results and suggestions for further work.

**Notation**

- **FMEDA**: failure modes, effects, and diagnostics analysis
- **PFD**: probability of failure on demand
- **PFDavg**: probability of failure on demand averaged over $[0, T_P]$  
- **PIF**: probability of initial failure; $1 - R(0)$
- **PRV**: pressure relief valve(s)
- $q_i$: percentage of proof tests resulting in failure in the $i$th interval
- $R(0)$: initial reliability; $1 - $PIF$; may be less than 1
- $R(t)$: reliability function for $t > 0$
- $T_i$: equivalent failure time associated with $q_i$
- $T_P$: length of a particular PRV time in-service until proof test
- $\lambda$: useful-life failure rate, a constant
- $\lambda(t)$: failure rate as a function of time; may be a constant

**Quantal Response Analysis**

The theory and practice of quantal response analysis as applied to proof test data are well described in [1] and [2]. Here we provide sufficient background to support understanding of the analysis in the remainder of the paper.

In quantal response analysis, the in-service operating times between installation and first proof test, or between re-installation and subsequent proof tests, for each PRV are arranged in ascending order without regard to the outcome of the test. The ascending operating times are then grouped into $m$ non-overlapping time intervals such that each interval includes a small number of proof tests that resulted in failure. This grouping process is not arbitrary and requires some understanding of the assumptions and mathematical details of quantal response analysis. For the $i$th interval, $i = 1, \ldots, m$, two quantities are calculated: $q_i$, the fraction of failed proof tests for the interval, and $T_i$, calculated as either the average operating time for all PRV in the interval or the average operating time for just the failed PRV in the interval. There is rarely a significant difference due to different methods of calculating $T_i$ and, in general, either method is acceptable.
Once the $q_i$ and $T_i$ calculations are complete, a plot is made of the points $-\ln(1-q_i)$ vs $T_i$ and two different curves are fit to these points. First, a power curve of the form $A t^n + B$ is fit to all of the quantal response points generated by a given data set. The curve fit is valid if $A > 0$ and $B \geq 0$. Normally, a valid curve will rise approximately linearly at first but then will reach a time when the ascent is more rapid. This time region of rapid ascent indicates the end of the useful life and the need for proof testing and maintenance. Furthermore, the value of $B$ from a valid curve fit is, for $B < 0.05$, an estimate of the PIF. Secondly, once the useful-life region is identified, the data points for just the useful life are fitted with a straight line of the form $mt + b$. The curve fit is valid if $m > 0$ and $b \geq 0$. The slope of the straight line, $m$, is an estimate for the constant useful-life failure rate, $\lambda$, given in failures/year. This can be converted to the more familiar failures/hour by dividing by 8760 hours/year. If $b < 0.05$, $b$ is another estimate for the PIF. The values of $B$ obtained for a power fit and $b$ obtained for a straight line fit should be similar for a given data set.

**Data Sets Analyzed**

Three independent data sets, obtained from two Fortune 500 operating companies, were analyzed by three independent analysis groups. Two of the analysis groups were from within the operating companies. The third analysis was independent of either operating company. The analyses produced the information contained in the Appendix Tables A1-A3. Data Set I consisted of 3403 proof tests performed on 1949 individual PRV resulting in 48 "fail-to-open" test results. Data Set II consisted of 2578 proof tests which included 57 failures. Data Set III consisted of 3282 proof tests which included 24 failures. Data Set III is unique in that the 3282 proof tests include 2377 proof tests that were performed prior to initial installation and these tests include 10 initial failures.

**Results of Data Analysis**

**Determining the Useful Life Interval**

Fitting power curves of the form $A t^n + B$ to the data in Tables A1-A3 results in the plots in Figure 1. The annotations in each graph indicate the estimated values for $A$, $n$, and $B$ with their standard deviations (for Data Sets I and II) along with the mean squared and root mean squared errors.
Figure 1. Plots of $-\ln(1-q)$ vs $T_i$ along with power curve fits for the three data sets.

First note, that in each case, $A$ and $B$ are positive, indicating valid curve fits. For Data Set I there is no significant increase in rate of ascent within the region for which we have data. This indicates that wear-out has not yet become evident at the time of the last data point at approximately 4.2 years. For Data Set II, a change in rate of ascent occurs between 4 and 5 years indicating the first evidence of wear-out. In Data Set III, although there is no significant change in rate of ascent until just after 7 years, it is also true that there are no data points between 5.2 years and 8.1 years to shape the curve. Absent this information, we must conservatively conclude that there is no evidence of wear-out prior to 5.2 years but beyond that, there is insufficient information to support a useful-life interval longer than about 5.2 years. Based on analysis of these three data sets, we conclude that PRV useful life extends to about 4 - 5 years at which time proof testing and maintenance are required to extend the PRV useful-life. If proof testing and maintenance are not performed by this time, we must consider the PRV to be beyond its useful life for purposes of reliability and risk analysis.

**Determining $\lambda$, the Useful-Life Constant Failure Rate**

Using linear regression, a straight line of the form $mt + b$ is fit to the useful-life data for each data set, resulting in the plots shown in Figure 2. The annotations in each graph indicate the estimated values for $m$ (in failures/year) and $b$ with their standard deviations along with the correlation coefficient, $r$, and the mean squared and root mean squared errors.
Figure 2. Plots of $-\ln(1-q)$ vs $T_i$ along with linear fits for the three data sets.

The values for $m$ are all of the same order of magnitude and, when divided by 8760 hour/year, correspond to failure rates between $10^{-8}$ and $10^{-7}$ failures/hour as summarized in Table 1 below. It is noted that the $r$ values are small and this may lead some readers to question the appropriateness of a linear fit to this data. However, as is explained in detail in [1] along with corroborating simulation evidence, the small values for $r$ result from the fact that the PIF is substantial and initial failures, in fact, account for most of the failures discovered in proof test. To further support the appropriateness of the linear regressions and the information derived from them, we performed a FMEDA analysis [3] on a particular PRV model. The FMEDA predicts a failure rate based on a database of failure rates [4] for each part used to construct the PVR. The database is designed to generate conservative estimates of PRV failure rate. For the specific PRV model analyzed, the FMEDA predicted a failure rate of $8.4\times10^{-8}$ failures/hour [1], consistent with the estimates for failure rates obtained from our analyses based on data aggregated over many PRV manufacturers and models.
Summary of Data Analysis Results

Table 1. Summary of Results along with General Conclusions Drawn

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Estimated Useful-Life Interval</th>
<th>Estimated PIF from $B$</th>
<th>Estimated PIF from $b$</th>
<th>Estimated λ failures/hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>4.2 years</td>
<td>1.13%</td>
<td>1.09%</td>
<td>$5.0 \times 10^{-8}$</td>
</tr>
<tr>
<td>II</td>
<td>4-5 years</td>
<td>1.64%</td>
<td>1.58%</td>
<td>$6.1 \times 10^{-8}$</td>
</tr>
<tr>
<td>III</td>
<td>5.2 years</td>
<td>1.27%</td>
<td>1.22%</td>
<td>$2.2 \times 10^{-8}$</td>
</tr>
<tr>
<td>General Conclusions</td>
<td>4 - 5 years</td>
<td>1 - 1.6%</td>
<td></td>
<td>$10^{-8} - 10^{-7}$</td>
</tr>
</tbody>
</table>

Note that for each data set the PIF estimated as $B$ from the power fit and as $b$ from the linear fit is about the same.

Incorporating PIF into PFD

PFD is a time varying quantity that measures the probability that the PRV is in a "stuck-shut" state when an over-pressure event occurs, i.e., the probability that the PRV is failed when a demand occurs. By definition,

$$PFD(t) = 1 - R(0)R(t).$$  \hspace{1cm} (1)

If the PRV is known to be in working order when the process is started, $R(0) = 1$. However, the data clearly show that $R(0) < 1$. To incorporate this into the calculation of PFD, we write

$$PFD(t) = 1 - (1 - PIF) R(t) = 1 - (1 - PIF) e^{-\lambda t}.$$ \hspace{1cm} (2)

For $\lambda$ as large as $10^{-7}$ and $t$ as large as 5 years (43,800 hours), $\lambda t$ is less than 0.005. Hence, $e^{-\lambda t}$ is well approximated by $(1 - \lambda t)$, allowing us to approximate (2) as

$$PFD(t) \approx 1 - (1 - PIF) (1 - \lambda t).$$ \hspace{1cm} (3)

Usually, we calculate not $PFD(t)$ but rather $PFD_{avg}$ defined as

0

$$PFD_{avg} = \frac{1}{T_P} \int_0^{T_P} PFD(t) \, dt$$ \hspace{1cm} (4)
where \( T_p \) is the in-service time of the PRV from last installation/reinstallation until the current proof test. Using (3) to approximate PFD(t) in (4) and integrating results in

\[
\text{PDFavg} \approx \text{PIF} + (1 - \text{PIF}) \times \lambda T_p / 2.
\]  

(5)

In (5) we see that the longer we wait before proof test, the greater the value of PDFavg.

To determine the extent to which PIF influences PDFavg, consider the ratio PIF/PDFavg. This ratio is plotted in Figure 3 for several values of PIF with \( \lambda = 10^{-7} \) over a range of \( T_p \) from 0 to 5 years. As would be expected, the PDFavg is initially due solely to the PIF and thus the ratio is 1. As time progresses without a proof test, random failures during the useful life of the PRV contribute to PDFavg and the ratio becomes smaller. However, note that for a PIF as little as 0.5% the ratio dips just under 0.7 after 5 years. A PIF of only 0.5% accounts almost 70% of the total PDFavg at 5 years. Clearly the PIF plays a strong role in reducing the safety that could otherwise be contributed by the PRV.

Figure 3. Plot of PIF/PDFavg for various values of PIF
Discussion and Suggestions for the Further Work

The main findings and implications of this research are appropriately captured in the ABSTRACT section at the top of this paper and are not repeated here. This section discusses a few additional points and suggests areas for further work.

FMEDA analysis is an appropriate tool for estimating useful-life failure rates for a given PRV model. It does not require extensive data collection and is the only viable method for predicting failure rates of new designs either not yet built or without sufficient numbers and in-service times to allow for data analysis. However, FMEDA analysis cannot predict PIF, an important parameter to be considered in risk assessment. Thus, this research clearly shows the need for adequate and appropriate data collection and analysis.

Given the dominant effect of PIF on the process safety provided by the PRV, it is crucial to identify and address the underlying causes of initial failures. If PIF can be significantly reduced, process safety can be significantly improved while reducing the frequency of (but not eliminating the need for) proof testing. Of course, to establish that PIF is, in fact, being reduced will require adequate and appropriate data collection and analysis.

References


Revision History

Authors: Dr. Julia V. Bukowski, Dr. William M. Goble

Acknowledgments

The authors wish to acknowledge the contributions and helpful suggestions of Harold Thomas of Air Products & Chemicals, Inc.
Appendix

The appendix contains the data tables generated by quantal response analysis of Data Sets I, II and III, which were reported in [1], [2], and [5] respectively. Note that in [5], the original data set included an interval with no failures. This is not permitted in quantal response analysis so the interval containing no failures was combined with the next interval to produce the data in Table A3 at \( T_i = 0.82 \) years.

**Table A1.** Quantal Response Data for Data Set I

<table>
<thead>
<tr>
<th>( T_i ) (years)</th>
<th>-ln(1-( q_i ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.65</td>
<td>0.0121</td>
</tr>
<tr>
<td>1.59</td>
<td>0.0102</td>
</tr>
<tr>
<td>1.96</td>
<td>0.0117</td>
</tr>
<tr>
<td>2.11</td>
<td>0.0136</td>
</tr>
<tr>
<td>2.42</td>
<td>0.0130</td>
</tr>
<tr>
<td>2.96</td>
<td>0.0042</td>
</tr>
<tr>
<td>3.28</td>
<td>0.0118</td>
</tr>
<tr>
<td>3.55</td>
<td>0.0169</td>
</tr>
<tr>
<td>3.74</td>
<td>0.0182</td>
</tr>
<tr>
<td>4.21</td>
<td>0.0090</td>
</tr>
</tbody>
</table>

**Table A2.** Quantal Response Data for Data Set II

<table>
<thead>
<tr>
<th>( T_i ) (years)</th>
<th>-ln(1-( q_i ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.16</td>
<td>0.0052</td>
</tr>
<tr>
<td>0.57</td>
<td>0.0072</td>
</tr>
<tr>
<td>0.79</td>
<td>0.0322</td>
</tr>
<tr>
<td>0.88</td>
<td>0.0120</td>
</tr>
<tr>
<td>0.93</td>
<td>0.0267</td>
</tr>
<tr>
<td>0.98</td>
<td>0.0080</td>
</tr>
<tr>
<td>1.03</td>
<td>0.0100</td>
</tr>
</tbody>
</table>
### Table A3. Quantal Response Data for Data Set III

<table>
<thead>
<tr>
<th>$T_i$ (years)</th>
<th>$-\ln(1-\hat{q}_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0042</td>
</tr>
<tr>
<td>0.82</td>
<td>0.0163</td>
</tr>
<tr>
<td>2.60</td>
<td>0.0244</td>
</tr>
<tr>
<td>3.15</td>
<td>0.0113</td>
</tr>
<tr>
<td>4.41</td>
<td>0.0087</td>
</tr>
<tr>
<td>5.18</td>
<td>0.0112</td>
</tr>
<tr>
<td>8.12</td>
<td>0.0392</td>
</tr>
</tbody>
</table>
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exida is one of the world’s leading accredited certification and knowledge companies specializing in automation system cybersecurity, safety, and availability. Founded in 2000 by several of the world’s top reliability and safety experts, exida is a global company with offices around the world. exida offers training, coaching, project-oriented consulting services, standalone and internet-based safety and cybersecurity engineering tools, detailed product assurance and certification analysis, and a collection of online safety, reliability, and cybersecurity resources. exida maintains a comprehensive failure rate and failure mode database on electrical and mechanical components, as well as automation equipment based on hundreds of field failure data sets representing over 350 billion unit operating hours.

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